# Signs of quadratic function

## Part 1

# Quadratic functions

**General form** : $f(x) = ax^2 + bx + c$

**Roots** : **According to the sign of the discriminant $\Delta$**

**Factorized form** : **According to the number of roots of the function**

| $\Delta > 0$ | $\Delta = 0$ | $\Delta < 0$ |
|---|---|---|
| ❖ Two distinct roots.<br>❖ $x = \frac{-b \pm \sqrt{\Delta}}{2a}$<br>❖ Factorized form: $a(x - x_1)(x - x_2)$ | ❖ One double root<br>❖ $x = -\frac{b}{2a}$<br>❖ Factorized form: $a(x - x_1)^2$ | ❖ No real roots<br>❖ $f(x)$ cannot be written in factorized form |

# Signs of quadratic functions

## Case 1: $\Delta > 0$

$f(x) = ax^2 + bx + c = a(x - x_1)(x - x_2)$

Suppose that $x_1 < x_2$

if $a > 0$

| $x$ | $-\infty$ | | $x_1$ | | $x_2$ | | $+\infty$ |
|---|---|---|---|---|---|---|---|
| $x - x_1$ | | $-$ | 0 | $+$ | | $+$ | |
| $x - x_2$ | | $-$ | | $-$ | 0 | $+$ | |
| $a$ | | $+$ | | $+$ | | $+$ | |
| $f(x)$ | | $+$ | 0 | $-$ | 0 | $+$ | |
| | | Same sign as $a$ | | opposite sign of a | | Same sign as $a$ | |

# Signs of quadratic functions

**Case 1: $\Delta > 0$**

$f(x) = ax^2 + bx + c = a(x - x_1)(x - x_2)$

Suppose that $x_1 < x_2$

if $a < 0$

| $x$ | $-\infty$ | $x_1$ | | $x_2$ | $+\infty$ |
|---|---|---|---|---|---|
| $x - x_1$ | $-$ | 0 | $+$ | | $+$ |
| $x - x_2$ | $-$ | | $-$ | 0 | $+$ |
| $a$ | $-$ | | $-$ | | $-$ |
| $f(x)$ | $-$ | 0 | $+$ | 0 | $-$ |
| | Same sign as $a$ | | opposite sign of a | | Same sign as $a$ |

# Signs of quadratic functions

## Case 1: $\Delta > 0$

$$f(x) = ax^2 + bx + c = a(x - x_1)(x - x_2)$$

Suppose that $x_1 < x_2$

General Rule: The signs of a quadratic function having two distinct solutions are summarized in the following table:

| $x$ | $-\infty$ | | $x_1$ | | $x_2$ | | $+\infty$ |
|---|---|---|---|---|---|---|---|
| $f(x)$ | | Same sign as $a$ | 0 | opposite sign of $a$ | 0 | Same sign as $a$ | |

S.O.S

# Signs of quadratic functions

## Case 1: $\Delta > 0$

Example 1: Study the sign of $f(x) = x^2 - 3x + 2$

$$\Delta = b^2 - 4ac = (-3)^2 - 4(1)(2) = 9 - 8 = 1 > 0$$

$$x_1 = \frac{-b-\sqrt{\Delta}}{2a} = \frac{3-1}{2} = 1 \quad ; \quad x_2 = \frac{-b+\sqrt{\Delta}}{2a} = \frac{3+1}{2} = 2$$

| $x$ | $-\infty$ | | $1$ | | $2$ | | $+\infty$ |
|---|---|---|---|---|---|---|---|
| $f(x)$ | | $+$ | $0$ | $-$ | $0$ | $+$ | |

# Signs of quadratic functions

**Case 1: $\Delta > 0$**

Example 2: Study the sign of $f(x) = -2x^2 + 5x - 3$

$$\Delta = b^2 - 4ac = 5^2 - 4(-2)(-3) = 25 - 24 = 1 > 0$$

$$x_1 = \frac{-b-\sqrt{\Delta}}{2a} = \frac{-5-1}{2(-2)} = \frac{3}{2} \quad ; \quad x_2 = \frac{-b+\sqrt{\Delta}}{2a} = \frac{-5+1}{2(-2)} = 1$$

| $x$ | $-\infty$ | | $1$ | | $\frac{3}{2}$ | | $+\infty$ |
|---|---|---|---|---|---|---|---|
| $f(x)$ | | $-$ | $0$ | $+$ | $0$ | $-$ | |

# Signs of quadratic functions

**Case 2: $\Delta = 0$**

$$f(x) = ax^2 + bx + c = a(x - x_1)^2$$

Positive for all values of $x$

Then, the signs of f(x) is always same as the real number $a$

| | |
|---|---|
| If $a > 0$ | • $f(x) \geq 0$ for all values of $x$ |
| If $a < 0$ | • $f(x) \leq 0$ for all values of $x$ |

# Signs of quadratic functions

## Case 2: $\Delta = 0$

$$f(x) = ax^2 + bx + c = a(x - x_1)^2$$

**General Rule:**

The sign of a quadratic function having one double root is summarized in the following table:

| $x$ | $-\infty$ | | $x_1$ | | $+\infty$ |
|---|---|---|---|---|---|
| $f(x)$ | | Same sign as $a$ | 0 | Same sign as $a$ | |

# Signs of quadratic functions

Case 2: $\Delta = 0$

Example 1: Study the sign of $f(x) = x^2 - 4x + 4$

$$\Delta = b^2 - 4ac = (-4)^2 - 4(1)(4) = 16 - 16 = 0$$

$$x_1 = x_2 = -\frac{b}{2a} = -\frac{-4}{2} = 2$$

| $x$ | $-\infty$ | | 2 | | $+\infty$ |
|---|---|---|---|---|---|
| $f(x)$ | | + | 0 | + | |

# Signs of quadratic functions

## Case 2: $\Delta = 0$

Example 2: Study the sign of $f(x) = -4x^2 - 12x - 9$

$$\Delta = b^2 - 4ac = (-12)^2 - 4(-4)(-9) = 144 - 144 = 0$$

$$x_1 = x_2 = -\frac{b}{2a} = -\frac{-12}{2(-4)} = \frac{3}{2}$$

| $x$ | $-\infty$ | | $\frac{3}{2}$ | | $+\infty$ |
|---|---|---|---|---|---|
| $f(x)$ | | $-$ | $0$ | $-$ | |

# Signs of quadratic functions

Case 3: $\Delta < 0$

$$f(x) = ax^2 + bx + c = a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right)$$

$$= a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2} + \frac{c}{a}\right)$$

$$= a\left(\left(x - \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a}\right) = a\left(\left(x - \frac{b}{2a}\right)^2 - \frac{b^2-4ac}{4a^2}\right)$$

$$= a\left(\left(x - \frac{b}{2a}\right)^2 - \frac{b^2-4ac}{4a^2}\right) = a\left(\left(x - \frac{b}{2a}\right)^2 - \frac{\Delta}{4a^2}\right)$$

# Signs of quadratic functions

## Case 3: $\Delta < 0$

$$f(x) = ax^2 + bx + c = a\left(\left(x - \frac{b}{2a}\right)^2 - \frac{\Delta}{4a^2}\right)$$

Negative

Positive

Positive

Then, the sign of f(x) is always same as the real number $a$

| | |
|---|---|
| If $a > 0$ | • $f(x) > 0$ for all values of $x$ |
| If $a < 0$ | • $f(x) < 0$ for all values of $x$ |

# Signs of quadratic functions

## Case 3: $\Delta < 0$

$f(x) = ax^2 + bx + c$

**General Rule:**

The sign of a quadratic function having no real roots is summarized in the following table:

| $x$ | $-\infty$ $\qquad$ $+\infty$ |
|---|---|
| $f(x)$ | Same sign as $a$ |

# Signs of quadratic functions

Case 3: $\Delta < 0$

Example 1: Study the sign of $f(x) = x^2 + 2x + 5$

$$\Delta = b^2 - 4ac = (2)^2 - 4(1)(5) = 4 - 20 = -16 < 0$$

no real roots.

| $x$ | $-\infty$ | $+\infty$ |
| --- | --- | --- |
| $f(x)$ | + | |

# Signs of quadratic functions

Case 3: $\Delta < 0$

Example 2: Study the sign of $f(x) = -x^2 + x - 3$

$$\Delta = b^2 - 4ac = (1)^2 - 4(-1)(-3) = 1 - 12 = -11 < 0$$

no real roots.

| $x$ | $-\infty$ $\qquad\qquad$ $+\infty$ |
|---|---|
| $f(x)$ | $-$ |

# Time for practice

Match

1) $f(x) = x^2 - 5x + 6$

2) $f(x) = -x^2 + 4x - 9$

3) $f(x) = -2x^2 + 12x - 18$

| $x$ | $-\infty$ | | $+\infty$ |
|---|---|---|---|
| $f(x)$ | | $-$ | |

| $x$ | $-\infty$ | | $3$ | | $+\infty$ |
|---|---|---|---|---|---|
| $f(x)$ | | $-$ | $0$ | $-$ | |

| $x$ | $-\infty$ | | $2$ | | $3$ | | $+\infty$ |
|---|---|---|---|---|---|---|---|
| $f(x)$ | | $+$ | $0$ | $-$ | $0$ | $+$ | |