

Signs of quadratic function

Part 1

Quadratic functions

General form : $f(x) = ax^2 + bx + c$

Roots : According to the sign of the discriminant Δ

Factorized form : According to the number of roots of the function

$$\Delta > 0$$

- ❖ Two distinct roots.
- ❖ $x = \frac{-b \pm \sqrt{\Delta}}{2a}$
- ❖ Factorized form:
 $a(x - x_1)(x - x_2)$

$$\Delta = 0$$

- ❖ One double root
- ❖ $x = -\frac{b}{2a}$
- ❖ Factorized form:
 $a(x - x_1)^2$

$$\Delta < 0$$

- ❖ No real roots
- ❖ $f(x)$ cannot be written in factorized form








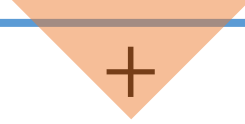
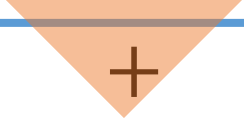



Signs of quadratic functions

Case 1: $\Delta > 0$

$$f(x) = ax^2 + bx + c = a(x - x_1)(x - x_2)$$

Suppose that $x_1 < x_2$

if $a > 0$

x	$-\infty$	x_1	x_2	$+\infty$
$x - x_1$		0		
$x - x_2$			0	
a				
$f(x)$		0		
	Same sign as a		opposite sign of a	Same sign as a

Signs of quadratic functions

Case 1: $\Delta > 0$

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Signs of quadratic functions

Case 1: $\Delta > 0$

$$f(x) = ax^2 + bx + c = a(x - x_1)(x - x_2)$$

Suppose that $x_1 < x_2$

General Rule: The signs of a quadratic function having two distinct solutions are summarized in the following table:

x	$-\infty$	x_1	x_2	$+\infty$	
$f(x)$	Same sign as a	0	opposite sign of a	0	Same sign as a

S.O.S




Signs of quadratic functions

Case 1: $\Delta > 0$

Example 1: Study the sign of $f(x) = x^2 - 3x + 2$

$$\Delta = b^2 - 4ac = (-3)^2 - 4(1)(2) = 9 - 8 = 1 > 0$$

$$x_1 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{3 - 1}{2} = 1 \quad ; \quad x_2 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{3 + 1}{2} = 2$$

x	$-\infty$	1	2	$+\infty$	
$f(x)$		0		0	

Signs of quadratic functions

Case 1: $\Delta > 0$

Example 2: Study the sign of $f(x) = -2x^2 + 5x - 3$

$$\Delta = b^2 - 4ac = 5^2 - 4(-2)(-3) = 25 - 24 = 1 > 0$$

$$x_1 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{-5 - 1}{2(-2)} = \frac{3}{2} \quad ; \quad x_2 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{-5 + 1}{2(-2)} = 1$$

x	$-\infty$	1	$\frac{3}{2}$	$+\infty$
$f(x)$	$-$	0	$+$	$-$

Signs of quadratic functions

Case 2: $\Delta = 0$

$$f(x) = ax^2 + bx + c = a(x - x_1)^2$$



Positive for all
values of x

Then, the signs of $f(x)$ is always same as the real number a

If $a > 0$

• $f(x) \geq 0$ for all values of x

If $a < 0$

• $f(x) \leq 0$ for all values of x

Signs of quadratic functions

Case 2: $\Delta = 0$

$$f(x) = ax^2 + bx + c = a(x - x_1)^2$$

General Rule:

The sign of a quadratic function having one double root is summarized in the following table:

x	$-\infty$	x_1	$+\infty$
$f(x)$	Same sign as a	0	Same sign as a

Signs of quadratic functions

Case 2: $\Delta = 0$

Example 1: Study the sign of $f(x) = x^2 - 4x + 4$

$$\Delta = b^2 - 4ac = (-4)^2 - 4(1)(4) = 16 - 16 = 0$$

$$x_1 = x_2 = -\frac{b}{2a} = -\frac{-4}{2} = 2$$

x	$-\infty$	2	$+\infty$
$f(x)$		$+$	$+$

Signs of quadratic functions

Case 2: $\Delta = 0$

Example 2: Study the sign of $f(x) = -4x^2 - 12x - 9$

$$\Delta = b^2 - 4ac = (-12)^2 - 4(-4)(-9) = 144 - 144 = 0$$

$$x_1 = x_2 = -\frac{b}{2a} = -\frac{-12}{2(-4)} = \frac{3}{2}$$

x	$-\infty$	$\frac{3}{2}$	$+\infty$
$f(x)$	—	0	—

Signs of quadratic functions

Case 3: $\Delta < 0$

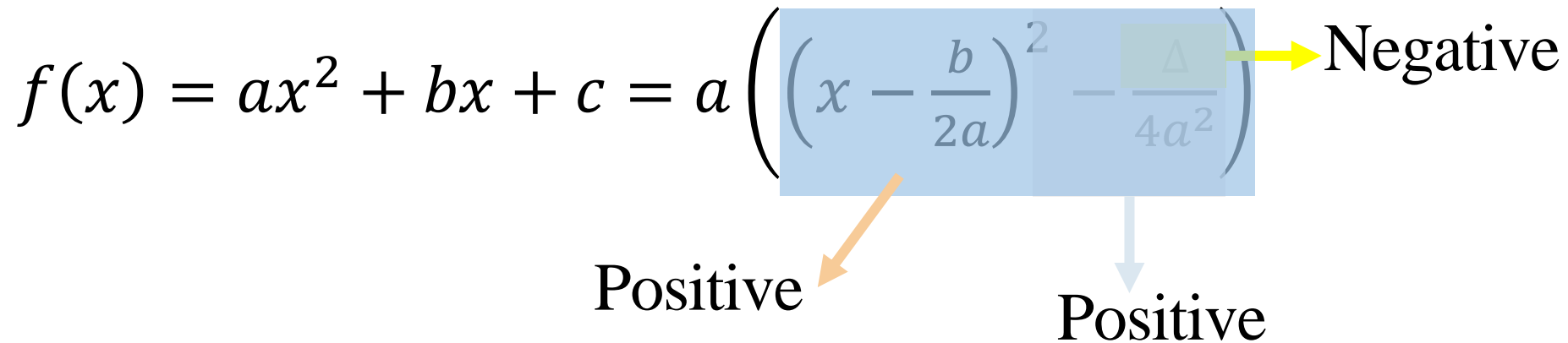
$$\begin{aligned} f(x) &= ax^2 + bx + c = a \left(x^2 + \frac{b}{a}x + \frac{c}{a} \right) \\ &= a \left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2} + \frac{c}{a} \right) \\ &= a \left(\left(x - \frac{b}{2a} \right)^2 - \frac{b^2}{4a^2} + \frac{c}{a} \right) = a \left(\left(x - \frac{b}{2a} \right)^2 - \frac{b^2 - 4ac}{4a^2} \right) \\ &= a \left(\left(x - \frac{b}{2a} \right)^2 - \frac{b^2 - 4ac}{4a^2} \right) = a \left(\left(x - \frac{b}{2a} \right)^2 - \frac{\Delta}{4a^2} \right) \end{aligned}$$

Signs of quadratic functions

Case 3: $\Delta < 0$

$$f(x) = ax^2 + bx + c = a \left(\left(x - \frac{b}{2a} \right)^2 - \frac{\Delta}{4a^2} \right)$$

Positive Positive Negative



Then, the sign of $f(x)$ is always same as the real number a

If $a > 0$

• $f(x) > 0$ for all values of x

If $a < 0$

• $f(x) < 0$ for all values of x

Signs of quadratic functions

Case 3: $\Delta < 0$

$$f(x) = ax^2 + bx + c$$

General Rule:

The sign of a quadratic function having no real roots is summarized in the following table:

x	$-\infty$	$+\infty$
$f(x)$	Same sign as a	

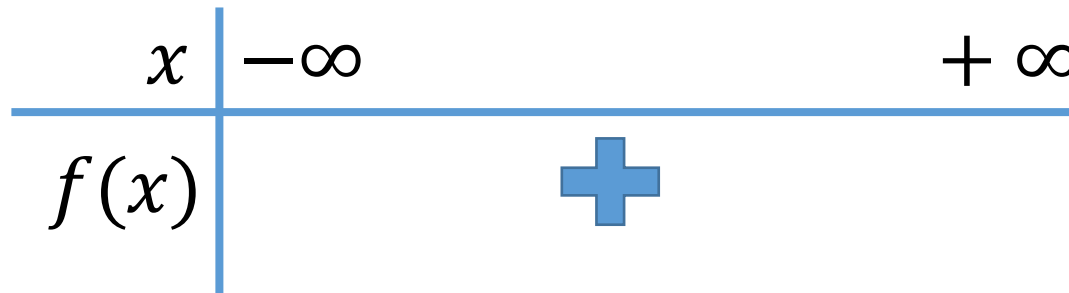
Signs of quadratic functions

Case 3: $\Delta < 0$

Example 1: Study the sign of $f(x) = x^2 + 2x + 5$

$$\Delta = b^2 - 4ac = (2)^2 - 4(1)(5) = 4 - 20 = -16 < 0$$

no real roots.



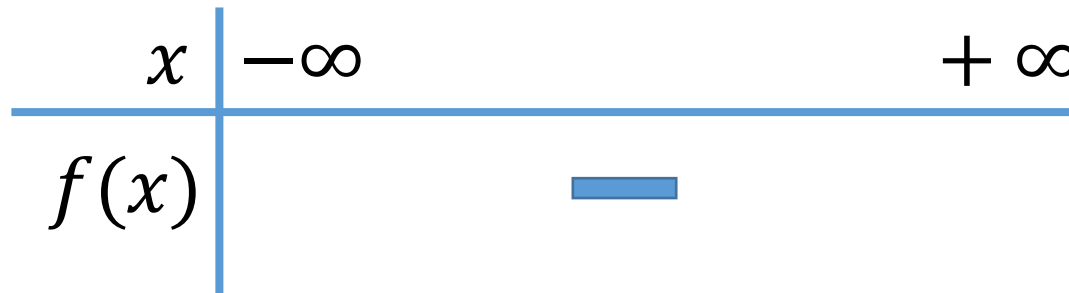
Signs of quadratic functions

Case 3: $\Delta < 0$

Example 2: Study the sign of $f(x) = -x^2 + x - 3$

$$\Delta = b^2 - 4ac = (1)^2 - 4(-1)(-3) = 1 - 12 = -11 < 0$$

no real roots.



Time for practice

Match

1) $f(x) = x^2 - 5x + 6$

2) $f(x) = -x^2 + 4x - 9$

3) $f(x) = -2x^2 + 12x - 18$

